How out of tune is Equal Temperament and how much do we care?
Bob Stuckey July 2020 version   public domain

The **tuning of the piano and all instruments of the orchestra** is based on the frets of the guitar, each fret raising the vibration by about 6% (also called a half-step, semitone or minor second). The 12 frets of the chromatic scale will take you up an octave. On the way up some of the tunings deviate slightly from the exact ratios between vibrations, e.g. 3/2, 5/4, hence the name *temperament*, that is, made impure. We can distinguish an Equally Tempered fret by slipping the initials ET into the word, thus frET.

In the examples that follow the amount of deviation is shown by dividing a frET into a hundred parts known as *cents*. In Equal Temperament ratios up to the number 4 are served very well with only 2 cents deviation. Though 3/2 and 4/3 are slightly out of tune it is not enough for any confusion to arise. As the numbers in the ratio get higher the intervals between them get gradually smaller, each time by a smaller amount, so that they can start to resemble each other. Most listeners, in a typical performance, are a happy to let **context** override **precision** tolerating deviations of up to 35 cents.

After the number 10 the intervals fall between two stools , somewhere between 1 and 2 frETs until you get to the ratio 17/18 which is 99% of the official frET. This was actually the ratio used by guitar makers before the mathematician Stevin came up with the slightly more elegant definition - the 12th root of 2.

A single frET is very distant from the other intervals. It is also different in that none of the first ten harmonics coincide, even nearly. It is more an **interval of convenience**, a unit that can, in combination, get close to the earlier intervals. It just happens to suit the delicate relationship between physics, biology and culture.

To illustrate how Equal Temperament approximates the simple ratios up to the number 10 cut out the shapes on the following page. Place them against the piano keyboard to demonstrate where the out-of-tuneness is and how flexible we can be when listening and how we can be influenced by context.

For example the notes D and B could pass for 6/5 with a G somewhere around but as 7/6 with an E and G# nearby (as part of an E7 chord) As shown below:

	G	B	D	F	
	4	5	6	7	*resembled harmonics*
4	5	6	7		*resembled harmonics*
E	G#	B	D		

The *diminished 7th* chord and the *7b9* chord owe their character to this ambiguity e.g.

	B	D	F	Ab	*are the 3 frETs between notes 6/5 or 7/6?*
G	B	D	F	Ab	

The notes F and B could pass for 7/5 with a G nearby (as part of a G7 chord) but as 10/7 with a Db nearby (as part of a Db7 chord). This harmonic device is known as the *tritone substitution.*

		G	B	D	F	
		4	5	6	7	*resembled harmonics*
	4			7	10	*resembled harmonics*
	Db		B		F	

The notes B and G could pass for 5/4 with a D around or 7/9 with an A, C# and E around (as part of an A9 chord).

			G	B	D	
			4	5	6	*resembled harmonics*
4	5	6	7	9		*resembled harmonics*
A	C#	E	G	B		

The notes F and its neighbour G could pass for:
8/7 with a B and D nearby (as part of a G7 chord) , or
9/8 with an A, C and Eb around (as part of an F9 chord) or as
10/9 with Eb, Db and Bb around (as part of and Eb9 chord)

		B	D	F	G	
		5	6	7	8	*resembled harmonics*
	5	6	7	8	9	*resembled harmonics*
	A	C	Eb	F	G	
		6	7	8	9	10 *resembled harmonics*
		Bb	Db	Eb	F	G

On the next page the resemblances between the ratios formed by pairs of harmonics between 5 and 10 are demonstrated by cut out shapes which can be placed against the piano keyboard.

A keyboard sized comparison of similar ratios and their nearest frET subsitute

frET=Equally tempered fret e.g. guitar
aka halfstep or semitone
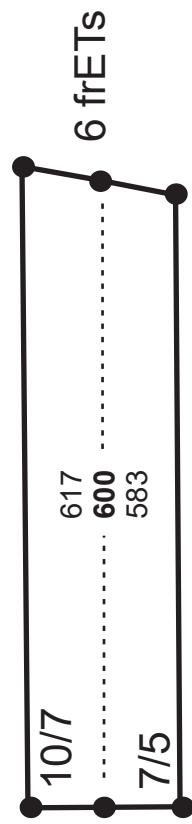
Print on paper or card and cut out the shapes.
Place the shapes against the back of the keyboard
where the black and white keys are of a more equal size.

ratio a/b

ratio a/b in cents

nearest frET in cents

ratio c/d in cents

ratio c/d

6 frETs

10/7
617
600
583
7/5

4 frETs

9/7
435
400
386
5/4

3 frETs

6/5
315
300
266
7/6

2 frETs

9/8 203
10/9 182
8/7 231
200

centimeters

0 1 2 3 4 5 6 7 8 9 10

1.375 centimeters per frET

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